(54) Title: METHOD AND SYSTEM FOR SOLVING FINITE ELEMENT MODELS USING MULTI-PHASE PHYSICS

(57) Abstract: Systems and methods for solving finite element models, wherein the matrix that governs the solution is modified by adjusting the weighting coefficients of the matrix so that the elements which lie on the diagonal of the matrix are non-negative and the elements which are off the diagonal are non-positive. In one embodiment, a system is discretized on a finite element mesh with the contribution of each node to the discretization being weighted based upon the direction of fluid flow across each element. The nodes which are upstream from the other nodes of the respective elements are weighted more heavily to cause the resulting matrix to be substantially diagonal. This matrix is solved using traditional techniques.

METHOD AND SYSTEM FOR SOLVING FINITE ELEMENT MODELS

USING MULTI-PHASE PHYSICS

Related Applications

[0001]   This application claims priority under 35 U.S.C. § 119(e) to United States Patent Application No. 60/215,697 entitled "Method and System for Oil Reservoir Simulation and Modeling" by Stephen R. Kennon, Kok Thye Lim, Scott A. Canaan, Steven B. Ward, Stuart W. Pond, Jr. and Edward J. Barragy, filed June 29, 2000, which is incorporated by reference as if set forth in its entirety herein.

Field of the Invention.

[0002]   The invention relates generally to methods for modeling physical systems using finite element analysis and, more specifically, to methods for solving finite element models wherein the matrix that controls the solution for the model is configured to maintain the properties of monotonicity and linearity preservation.

Background of the Invention.

[0003]   Physical systems can be modeled mathematically to simulate their behavior under certain conditions. There are a wide variety of means to model these systems, ranging from the very simplistic to the extremely complicated. One of the more sophisticated means to model physical systems is through the use of finite element analysis. As the name implies, finite element analysis involves the representation of individual,

finite elements of the physical system in a mathematical
model and the solution of this model in the presence of
a predetermined set of boundary conditions. Another
comparable means to model physical systems is through
the use of finite difference analysis. Finite
difference analysis involves the modeling of individual
points within a modeled space and computing the
differences between these points. Finite difference
analysis is often used for simulating the dynamic
behavior of fluids.

[0004]   Traditional finite difference techniques and finite
element techniques using streamlined, upwinding methods
are typically used to simulate the production of oil in
a reservoir. While each of these techniques has its own
advantages, it also has its own disadvantages.
Generally speaking, the finite difference techniques
produce physically realistic results, but they are not
very accurate. The finite element techniques, on the
other hand, are more accurate, but they produce results
which are not physically realistic.

[0005]   As a result of their respective disadvantages, both
the finite difference techniques and finite element
techniques which are conventionally used normally
require a great deal of computer resources. In the case
of finite difference techniques, reasonable accuracy can
be achieved, but this requires many more nodes then
would be necessary in a finite element model. This
increases the amount of memory and CPU time which are
needed to compute an accurate solution. In the case of
finite element techniques, comparable accuracy can be
achieved with fewer nodes, but the solutions may not be

realistic. For example, concentrations may be more than 100 percent, or permeabilities may be negative. Consequently, there may be problems for which the solution does not converge, or for which the solution may converge very slowly. These techniques are therefore less reliable and may require a large number of iterations before acceptable accuracy is achieved.

[0006] Because each of these standard techniques has its own drawbacks, and because these drawbacks increase in the amount of computer resources which are necessary to generate acceptable solutions, it would be desirable to provide a method for modeling systems such as oil reservoirs which reliably produces accurate, realistic solutions for these systems.

Summary of the Invention

[0007] One or more of the problems outlined above may be solved by the various embodiments of the invention. Broadly speaking, the invention comprises systems and methods for solving finite element models, wherein the matrix that governs the solution is modified so that it satisfies the properties of monotonicity and preservation of linearity. This is accomplished by adjusting the weighting coefficients of the matrix so that the elements which lie on the diagonal of the matrix are non-negative and the elements which are off the diagonal are non-positive. This causes the solution to be both more accurate than traditional finite difference techniques and more realistic than traditional finite element techniques.

[0008]   The present techniques may provide various
          advantages over the prior art. For instance, because
          the present techniques are more accurate, the number of
          iterations which are required to produce an acceptable
          result may be reduced, thereby conserving computing
          resources. The present techniques may also cause
          solutions to more reliably converge, since they are
          constrained to the range of realistic results and do not
          oscillate in the same manner as the solutions of
          traditional finite element techniques. The present
          techniques are therefore generally faster, more
          accurate, less resource-intensive and more reliable than
          prior art techniques.

[0009]   One embodiment of the present invention comprises a
          method for generating a solution for a modeled system
          using finite element techniques, wherein the system is
          discretized on a finite element mesh and wherein the
          contribution of each node to the discretization is
          weighted based upon the direction of fluid flow across
          each element. The contributions are weighted to favor
          the nodes which are upstream from the other nodes of the
          respective elements. Weighting the contributions of the
          nodes in this manner causes the resulting matrix to be
          essentially diagonal. That is, the elements which lie
          on the diagonal of the matrix are emphasized, while the
          elements which are off the diagonal are de-emphasized.
          Once the matrix has been modified in this manner, it is
          solved using traditional techniques.

[0010]   An alternate embodiment of the present invention
          comprises a method for modeling the flow of multiphase
          fluids in an oil reservoir. In this embodiment, a four

dimensional finite element mesh corresponding to the oil reservoir is constructed. A solution is generated for the mesh by distributing the residuals of a finite element operator among the nodes of the mesh and then assembling and solving the system using traditional finite element techniques. In this embodiment, distributing the residual of the finite element operator comprises, for each element, computing the value of the finite element operator for each node, dividing these values into a set of positive values and a set of negative values, distributing the residual to the nodes to emphasize the diagonal elements and de-emphasize the off-diagonal elements and then computing a tangent operator for the matrix.

[0011]   Another alternate embodiment of the present invention comprises a software product which is embodied in a computer readable medium. The software product comprises a plurality of instructions which are configured to cause a computer or any other data processor to perform the steps of a method in accordance with the present disclosure. The computer readable medium may comprise any of a number of different media, including but not limited to CD ROMs, DVDs, floppy disks, computer memories, magnetic tapes, etc. Still another alternate embodiment of the present invention comprises a computer system which is configured to perform the steps of a method in accordance with the present disclosure. The computer system may be programmed to perform such a method by instructions which are contained in hardware, firmware or software.

[0012]   Numerous additional alternative embodiments are also
          possible.

Brief Description of the Drawings

[0013]   Other objects and advantages of the invention may
          become apparent upon reading the following detailed
          description and upon reference to the accompanying
          drawings.

[0014]   FIGURE 1 is a diagram illustrating simplices in two
          and three dimensions.

[0015]   FIGURE 2 is a diagram illustrating the difference
          between the actual values in the physical system and the
          solutions generated using traditional finite element
          techniques and techniques according to the present
          invention.

[0016]   FIGURE 3 is a diagram illustrating the flow of fluid
          across a first two-dimensional simplex element.

[0017]   FIGURE 4 is a diagram illustrating the flow of fluid
          across the first two-dimensional simplex element in a
          direction opposite the flow illustrated in FIGURE 3.

[0018]   FIGURE 5 is a diagram illustrating the flow of fluid
          across a second two-dimensional simplex element.

[0019]   FIGURE 6 is a diagram illustrating the flow of fluid
          across the second two-dimensional simplex element in a
          direction opposite the flow illustrated in FIGURE 5.

[0020]   FIGURE 7 is a diagram illustrating a two-dimensional
          projection of a time-space domain.

[0021]   FIGURE 8 is a flow diagram illustrating a method in accordance with one embodiment of the present invention.

[0022]   While the invention is subject to various modifications and alternative forms, specific embodiments thereof are shown by way of example in the drawings and the accompanying detailed description. It should be understood, however, that the drawings and detailed description are not intended to limit the invention to the particular embodiment which is described. This disclosure is instead intended to cover all modifications, equivalents and alternatives falling within the scope of the present invention as defined by the appended claims.

Detailed Description of a Preferred Embodiment

[0023]   A preferred embodiment of the invention is described below. It should be noted that this and any other embodiments described below are exemplary and are intended to be illustrative of the invention rather than limiting.

[0024]   Broadly speaking, the invention comprises methods for solving finite element models, wherein the solution is guaranteed both to be monotonic and to satisfy linearity preservation. This ensures that the solution will be both accurate and realistic. The monotonicity and the linearity of the solution are ensured by generating a matrix for the solution of the finite element model such that the elements of the matrix which are on the diagonal are non-negative, and the elements which are off-diagonal are non-positive. This matrix can then be solved using conventional finite element

techniques to generate the desired accurate, realistic
solution.

[0025]   The present methods provide a number of advantages
over traditional finite element and finite difference
techniques. As indicated above, the present methods
provide the accuracy of traditional finite element
techniques without the disadvantages of unrealistic,
oscillating solutions. Similarly, the present methods
provide the stability of traditional finite difference
techniques while providing much greater accuracy.

[0026]   The accuracy and realism provided by the present
methods lead to additional advantages in the utilization
of computer resources. For example, the present methods
may allow for faster generation of solutions for
physical systems. This is a result of several things.
First, because the solution is physically realistic, it
does not oscillate, and consequently tends to converge
more rapidly. Further, because generation of the
solution using the present methods typically requires
less iterations and is faster than traditional finite
element techniques, less CPU time is required to
generate the solution. The present methods may also use
less memory than traditional techniques. Because the
present methods are generally more accurate than
traditional finite difference techniques, is not
necessary to use as many nodes to model the physical
system. Consequently, less memory is required to store
the model (and less CPU time is required to compute a
solution based on these nodes.) the present methods may
also be more reliable than traditional techniques. The
oscillation of solutions generated using traditional

finite element techniques may, in some instances,
prevent a solution from converging at all. As noted
above, the present methods provide solutions which are
realistic (e.g., they do not oscillate between realistic
solutions and solutions which are not possible,) so the
solutions will reliably converge.

[0027]   In finite element modeling, the region that is to be
analyzed is broken up into sub-regions called elements.
This process of dividing the region into sub-regions may
be referred to as discretization or mesh generation.
The region is represented by functions defined over each
element. This generates a number of local functions
that are much simpler than those which would be required
to represent the entire region. The next step is to
analyze the response for each element. This is
accomplished by building a matrix that defines the
properties of the various elements within the region and
a vector that defines the residuals acting on each
element in the domain. Once all the element matrices
and vectors have been created, they are combined into a
matrix equation. After applying boundary conditions, the
matrix equation can be solved to obtain unknown nodal
responses. Intra-element responses can be interpolated
from nodal values using the functions which were defined
over each element.

[0028]   The following description of one embodiment of the
present method relates to the modeling of oil
reservoirs. This embodiment is intended to be
exemplary, and it should be noted that the present
methods may be applied to a variety of physical systems,
such as flow over aircraft, weather prediction, flow

through internal combustion engines, and other
industrial fluid flows. This list of potential
applications of the present methods is intended to be
illustrative rather than limiting.

[0029]   The details of a preferred embodiment will be set
forth below. It may be helpful, however, to first
define a few terms.

[0030]   A node is a point in space. In finite element
modeling, nodes form the vertices of the elements which
are modeled. The nodes also form part of a mesh of
nodes and edges which define the boundaries between
elements in the modeled space.

[0031]   An edge is a line between two nodes which form
vertices of an element. The edges form part of the mesh
which defines the boundaries between elements in the
modeled space.

[0032]   A simplex is a spatial configuration of n dimensions
determined by n + 1 points in a space of dimension equal
to or greater than n. In other words, a—simplex is a
geometric spatial element having the minimum number of
boundary points necessary to enclose a space in a given
number of dimensions. For example, in two dimensions,
a simplex comprises a triangle, together with the
interior area bounded by the triangle (see FIGURE 1.)
Two points are insufficient to form a simplex in two-
dimensional space because no area is bounded by the
points (and the lines which interconnect them.) While
four points may be sufficient to bound a two-dimensional
area, they do not comprise the minimum number of
boundary points by which the two-dimensional area can be

bounded. In three dimensions, a simplex comprises a tetrahedron, which is bounded by four vertices (see FIGURE 1.) In four dimensions, a simplex comprises a hypertetrahedron (sometimes referred to as a hypertet) having five vertices.

[0033] A mesh is a collection of elements that fill a space. These elements are representative of a system which resides in that space. Because each element can be defined by a plurality of nodes and/or the edges between those nodes, a mesh may alternatively be considered a collection of nodes and/or the edges between them. At various points in this disclosure, "mesh" will be used to alternately refer to collections of elements or nodes/edges, depending upon the context in which the term is used. The mesh may also be referred to herein as a finite element model or simply a model.

[0034] There is a fundamental difference between modeling systems in which there is a single fluid phase and those in which there are multiple phases. When there are multiple fluid phases, there is a convection and potentially a mixing of the phases. (It is assumed that there is a pressure which drives the fluids and causes the front between them to be transported.) The finite element model must therefore include convective operators or components. These operators would not be necessary if only one fluid phase were modeled, or if the multiple fluid phases were completely mixed or emulsified.

[0035]   In a system having multiple fluid phases,
         characteristics of the system may change very abruptly.
         This situation is illustrated in FIGURE 2. FIGURE 2 is
         a diagram illustrating the difference between the actual
         values in the physical system and the solutions
         generated using traditional finite element techniques
         and techniques according to the present invention. It
         can be seen that the actual ("real world") values 11
         form what is essentially a step function, changing from
         a first value v1 to a second value v0 immediately upon
         reaching a distance d1. In this example, the values v0
         and v1 define the limits of the range of physically
         realizable values.

[0036]   When traditional finite element techniques are used
         to model such a system, the solution will typically tend
         to oscillate. It can be seen in FIGURE 2 that the
         solution 12 generated using these techniques oscillates
         above and below the actual physical values which it is
         attempting to model. Two of the most significant
         problems with this solution are the obvious inaccuracy
         (the solution does not closely track the actual values)
         and the fact that, at some points, the solution is
         greater than the maximum physically realizable value or
         less than the minimum physically realizable value. The
         oscillations which occur in solutions generated using
         traditional finite element techniques may therefore
         produce unrealistic results, or may even fail to
         converge at all.

[0037]   Methods in accordance with this disclosure, however,
         do not suffer from these disadvantages because the
         solutions are constrained to be realistic. This

constraint is achieved by ensuring that the matrix which
governs the solution vector (i.e., the Jacobian matrix)
satisfies the properties of monotonicity and
preservation of linearity. In more practical terms,
this consists of ensuring that all of the coefficients
on the diagonal of the matrix are non-negative and that
all of the coefficients off the diagonal of the matrix
are non-positive. When the matrix has these
characteristics, the solution remains realistic at all
points, and more accurately tracks the behavior of the
physical system, as indicated by curve 13 in FIGURE 2.
The model is therefore more likely to produce accurate
results and to reliably converge.

[0038]   The Jacobian matrix is derived from the partial
differential equations that describe multiphase flow
through porous media based on Darcy's law, and
representing conservation of mass and momentum in the
system. These equations are discretized onto the finite
element mesh, and the coefficients of the equations form
a matrix. Each coefficient will be either positive or
negative, depending upon whether the gradient of the
equation is positive or negative (i.e., whether the
fluid flow is positive or negative). If the finite
element technique is applied to the differential
equations, a large set of algebraic equations is
produced. That set of algebraic equations is nonlinear.
The set of equations can be solved by using Newton's
method (which involves taking the first derivative of
the equations). The first derivative is the Jacobian
matrix. The coefficients of this matrix are weighted to
obtain a matrix which has non-negative terms on its

diagonal and non-positive terms off the diagonal. The
appropriate weighting of the terms ensures the monotonic
and linear properties of the matrix.

[0039]   The coefficients of the Jacobian matrix are
essentially weighting factors for each of the nodes of
the elements in the finite element mesh. In traditional
finite element techniques, the nodes of an element are
evenly weighted. For example, in a two-dimensional
element, each of the three nodes would be weighted by
1/3. If a three-dimensional element were used, each of
the four nodes of the element would be weighted by 1/4.
In the present methods, however, the nodes are weighted
to reflect the flow of the fluids in the modeled system.
This is illustrated with reference to FIGURES 3 - 6.

[0040]   FIGURE 3 shows a single, two-dimensional element (a
triangle). The flow of fluid with respect to the
element is indicated by the arrows, which show that the
fluid is flowing from the upper, right-hand portion of
the figure to the lower, left-hand portion of the
figure. Using traditional finite element techniques,
each of nodes 21 - 23 would be evenly weighted (i.e.,
1/3, 1/3, 1/3). Using the present techniques, the nodes
are not evenly weighted. Because the fluid is flowing
generally from nodes 22 and 23 to node 21, each of nodes
22 and 23 is weighted by 1/2, while node 21 is weighted
by 0. Referring to FIGURE 4, it can be seen that the
weighting of the nodes is different when the direction
of the fluid flow changes. In FIGURE 4, the fluid flow
is generally from the lower, left-hand portion of figure
to the upper, right-hand portion of the figure.

Consequently, node 21 is weighted by 1, while nodes 22 and 23 are weighted by 0.

[0041]   Referring to FIGURES 5 and 6, it can be seen that the weighting of the nodes is also affected by the shape of the corresponding element. In FIGURE 5, the flow of fluid is approximately perpendicular to an edge of the element. This situation is similar to that of FIGURE 3, in which each of the nodes which terminates the forward edge of the element was weighted by 1/2. In that case, the remaining node of the element was downstream from a midway point on the forward edge of the element. In FIGURE 5, the remaining node is instead directly downstream from one of the first to nodes. In this situation, the node 32 which is upstream from the last node 33 is weighted by 2/3, while the other upstream node 31is weighted by 1/3. In FIGURE 6, there is a single upstream node 33. Nodes 31 and 32 are both on the trailing edge of the element, so they are both weighted by 0. The upstream node 33, the other hand, is weighted by 1.

[0042]   In each of these cases, the weighting of the nodes with respect to the direction of fluid flow effectively emphasizes the diagonal elements of the matrix, while reducing or eliminating the effect of the off-diagonal elements. This, in turn, ensures that the matrix has the properties of monotonicity and linearity.

[0043]   Although the foregoing is sufficient to enable a person of ordinary skill in the field of finite element analysis to implement the present techniques, a brief technical discussion of the techniques follows.

[0044]   Considering a physical system which is to be modeled over time, a finite element mesh is first constructed (i.e., the system is discretized in the space-time domain). Referring to FIGURE 7, a two-dimensional projection of the time-space domain is illustrated.

[0045] $\Omega = \Omega_{space} \times [0, T]$ is the space-time domain to be modeled, where $\Omega_{space} \in \mathfrak{R}^3$.

[0046] Ω is broken down (segmented) into a plurality of non-overlapping sub-domains Ω_e. Each of these sub-domains Ω_e and $\bigcup \overline{\Omega_e} = \overline{\Omega}$ $\quad e \in \{1, ..., N_E\}$ comprises a four-dimensional simplex (a hypertetrahedron). A linear finite element basis is formed on each element (Ω_e), and trial functions and test functions are defined on each element.

[0047]   The flow of multi-phase fluids in porous media is governed by the equation:

$$\frac{\partial}{\partial t}\left(\phi\frac{S_i}{B_i}\right) + \nabla \bullet (V_i \lambda_i) = 0$$

Where:

$$i = 1, 2, ..., \text{nphases}$$

$$V_i = -K(\nabla P_i + \rho_i g \nabla Z)$$

$$P_1 = P$$

$$P_i = P - P_{c_i}, \, i=2, ..., \text{nphases}$$

$$\lambda_i = \frac{K_{r_i}}{\mu_i B_i}$$

$$\mu_i = \text{Viscosity of } i'\text{th phase}$$

$$B_i = \text{Formation Volume Factor of } i'\text{th phase}$$

$$\rho_i = \text{Density of } i'\text{th phase}$$

$$\phi = \text{Rock Porosity}$$

$$Z = \text{Negative of Depth}$$

$$S_i = \text{Saturation of } i'\text{th phase}$$

$$\sum_{i}^{\text{nphase}} S_i = 1$$

[0048]

[0049]   A standard Galerkin Finite Element discretization of

$$(\tilde{T}_j^i) = \int_{\Omega_e} N_j \frac{\partial}{\partial t}\left(\left(\frac{\Phi S_i}{B_i}\right)_k N_k\right) + \left[-\int_{\Omega_e} \nabla N_j \bullet \underset{\sim}{K}(\nabla P_i + \rho_i g \nabla Z)\Big|_{\Omega_e} \overline{(\lambda_i)\Big|_{\Omega_e}}\right]$$

this multi-phase flow equation gives, for each element:

[0050]

[0051]   The basic method of employing the present techniques

is defined by the way that $\overline{\lambda}_i$ is defined. The present

method is therefore defined as follows:

$$\tilde{\Upsilon}_j^{\ i} = \left(\int_{\Omega_e} N_j \frac{\partial}{\partial t} N_k + | \int_{\Omega_e} N_j \frac{\partial}{\partial t} N_k | \right) \left(\frac{\Phi S_i}{B_i} \right)_k + (\Upsilon_j^{\ i})_{spatial}$$

[0052]

[0053]   Referring to FIGURE 8, a flow diagram illustrating a
method in accordance with one embodiment of the present
invention is shown. As indicated above, when the
differential equations are discretized on the finite
elements, they are basically converted from differential
equations to algebraic relationships between the values
of the nodes. This is accomplished by assuming that
there is a linear variation of the corresponding
property (e.g. saturation, pressure, permeability,
prostate, etc.) between the nodes. This restricts the
space of all possible solutions to only those solutions
which are linear. When this assumption is used, a set
of relations are derived from the differential equations
for values of the coefficients which are linear over
each element. When this is done, each element
contributes a certain amount of the discretization to
each one of its nodes. The contributions which are
summed at each node are the residuals. It is desired to
make the residuals equal 0 in order to solve the
nonlinear algebraic system.

[0054]   To distribute the residuals of the finite element
operator ($\Upsilon_j^{\ i}$), K_j is first computed for each node of the
4-tet (hypertetrahedron). The resulting K_j are divided
into positive (K_j^+) and negative (K_j^-) values. The
residual Υ_j is then distributed to the nodes, and the
tangent operator (T_{jk}) is computed. The system is then
assembled and solved in the same manner as if an

ordinary finite element technique had been used.   (Since
the process by which finite element systems are
assembled and solved is well known, it will not be
described in further detail here).

[0055]   The benefits and advantages which may be provided by
the present invention have been described above with
regard to specific embodiments.   These benefits and
advantages, and any elements or limitations that may
cause them to occur or to become more pronounced are not
to be construed as a critical, required, or essential
features of any or all of the claims.   As used herein,
the terms "comprises," "comprising," or any other
variations thereof, are intended to be interpreted as
non-exclusively including the elements or limitations
which follow those terms.   Accordingly, a process,
method, article, or apparatus that comprises a list of
elements does not include only those elements but may
include other elements not expressly listed or inherent
to the claimed process, method, article, or apparatus.

[0056]   While the present invention has been described with
reference to particular embodiments, it should be
understood that the embodiments are illustrative and
that the scope of the invention is not limited to these
embodiments.   Many variations, modifications, additions
and improvements to the embodiments described above are
possible.   Particularly, these variations may include
computers or other data processing devices, computer
readable media (such as floppy disks, CD-ROMs, DVD-ROMs,
etc.,) storage devices, computer memories and the like
which contain software, firmware or other programming
embodying the foregoing methods.   It is contemplated

that these variations, modifications, additions and improvements fall within the scope of the invention as detailed within the following claims.

What is claimed is:

1.   A method for solving a finite element model corresponding to a system in which there is a multi-phase fluid flow comprising:

  generating a finite element matrix corresponding to the
      model, wherein the matrix contains a plurality of
      coefficients;

  adjusting the coefficients to obtain a matrix in which
      on-diagonal elements are non-negative and off-
      diagonal elements are non-positive; and

  generating a solution for the model using the matrix
      using finite element techniques.

2.   The method of claim 1 wherein adjusting the coefficients comprises weighting the nodes of each element according to a direction of fluid flow across the element.

3.   The method of claim 2 wherein weighting the nodes of each element according to a direction of fluid flow across the element comprises determining the direction of fluid flow across the element and weighting each node more heavily if the node is upstream from the other nodes of the element and less heavily if the node is downstream from the other nodes of the element.

4.   The method of claim 3 wherein each node is weighted more heavily if a greater portion of the element is downstream from the node than from other nodes of the element and less heavily if a smaller portion of the element is downstream from the node than from other nodes of the element.

5. The method of claim 1 wherein the finite element matrix corresponds to a system in which there are at least two fluid phases.

6. The method of claim 1 wherein the finite element matrix corresponds to a system in which there are three or more fluid phases.

7. The method of claim 1 wherein the finite element matrix corresponds to a four-dimensional finite element model.

8. The method of claim 1 wherein the system corresponds to an oil reservoir.

9. The method of claim 1 wherein the matrix is configured to produce a solution which is not physically unrealistic at any time.

10. The method of claim 1 wherein the matrix is configured to produce a solution which is non-oscillating.

11. The method of claim 1 further comprising discretizing a model of the system to produce a finite element mesh and generating the matrix based on the finite element mesh.

12. A method for obtaining improved accuracy in solving finite element models comprising:
    discretizing a model of a system in which there is a
        multi-phase fluid flow;

generating a finite element matrix corresponding to the
    model, wherein the matrix is configured to produce a
    solution which is monotonic and preserves linearity;
    and

generating a solution for the model using the matrix.

13.   A computer-readable medium which contains a plurality of
instructions, wherein the instructons are configured to cause
a computer to perform the method for solving a finite element
model corresponding to a system in which there is a multi-
phase fluid flow comprising:

generating a finite element matrix corresponding to the
    model, wherein the matrix contains a plurality of
    coefficients;

adjusting the coefficients to obtain a matrix in which
    on-diagonal elements are non-negative and off-
    diagonal elements are non-positive; and

generating a solution for the model using the matrix
    using finite element techniques.

14.   The computer-readable medium of claim 13 wherein
adjusting the coefficients comprises weighting the nodes of
each element according to a direction of fluid flow across the
element.

15.   The computer-readable medium of claim 14 wherein
weighting the nodes of each element according to a direction
of fluid flow across the element comprises determining the
direction of fluid flow across the element and weighting each
node more heavily if the node is upstream from the other nodes
of the element and less heavily if the node is downstream from
the other nodes of the element.

16.   The computer-readable medium of claim 15 wherein each node is weighted more heavily if a greater portion of the element is downstream from the node than from other nodes of the element and less heavily if a smaller portion of the element is downstream from the node than from other nodes of the element.

17.   The computer-readable medium of claim 13 wherein the finite element matrix corresponds to a system in which there are at least two fluid phases.

18.   The computer-readable medium of claim 13 wherein the finite element matrix corresponds to a system in which there are three or more fluid phases.

19.   The computer-readable medium of claim 13 wherein the finite element matrix corresponds to a four-dimensional finite element model.

20.   The computer-readable medium of claim 13 wherein the system corresponds to an oil reservoir.

21.   The computer-readable medium of claim 13 wherein the matrix is configured to produce a solution which is not physically unrealistic at any time.

22.   The computer-readable medium of claim 13 wherein the matrix is configured to produce a solution which is non-oscillating.

23. The computer-readable medium of claim 13 wherein the method further comprises discretizing a model of the system to produce a finite element mesh and generating the matrix based on the finite element mesh.
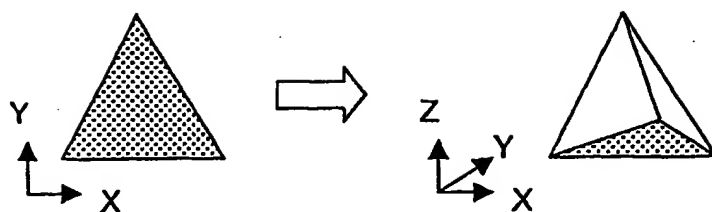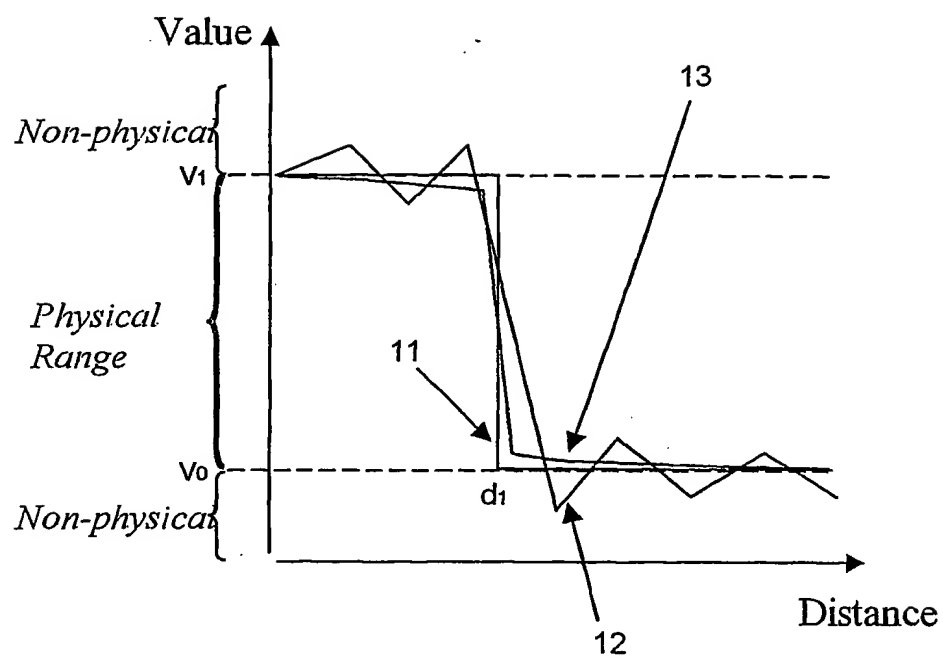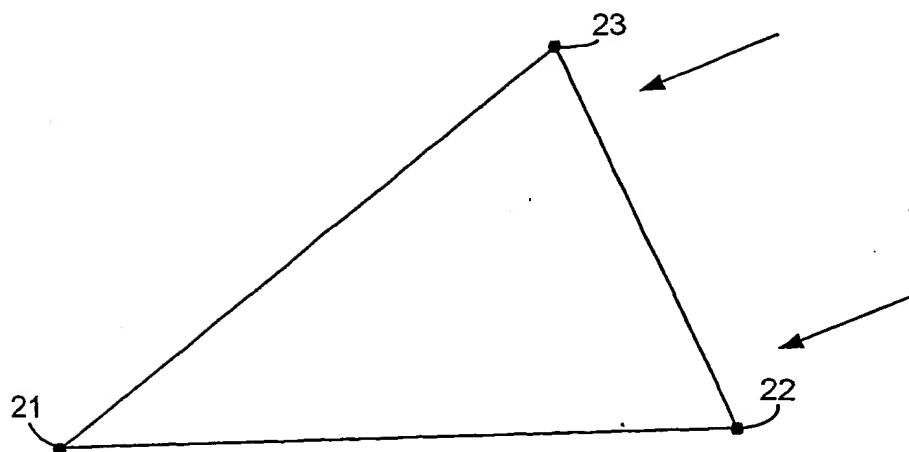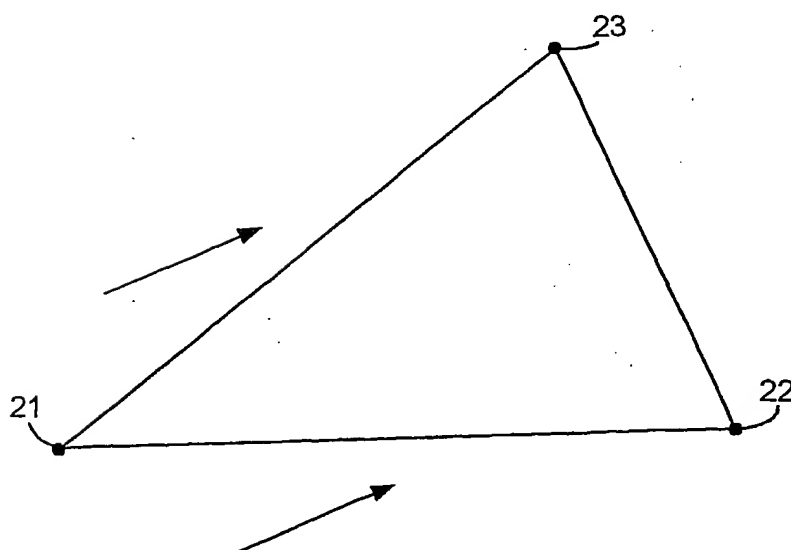
Fig. 1

Fig. 2

Fig. 3

Fig. 4
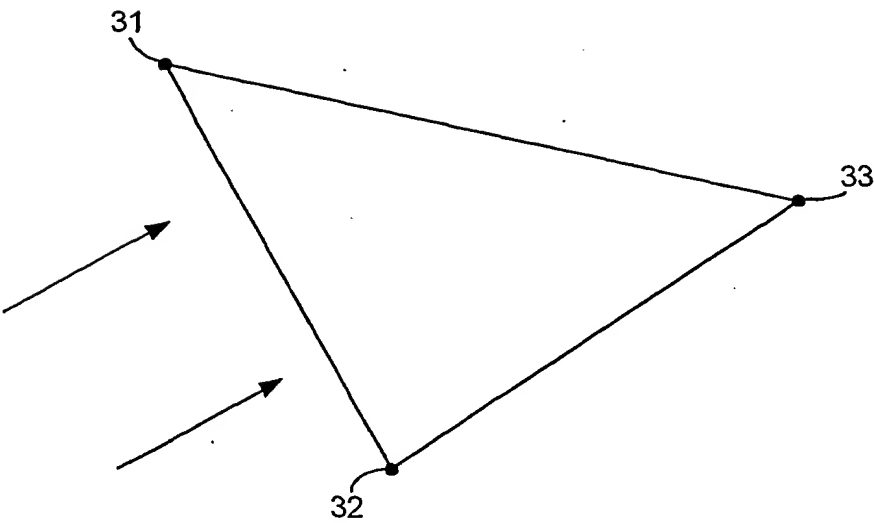
Fig. 5
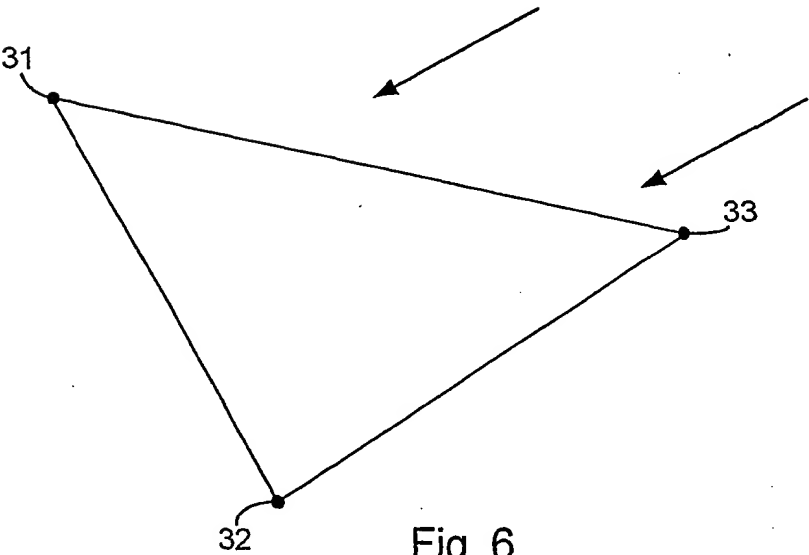
Fig. 6
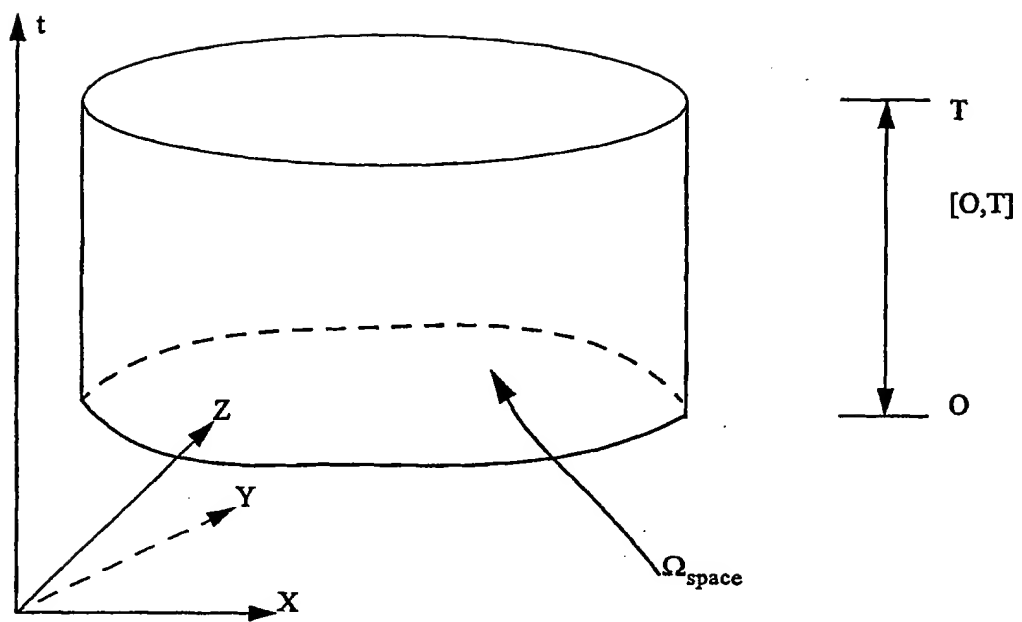
Fig. 7

Multi-Phase Flow
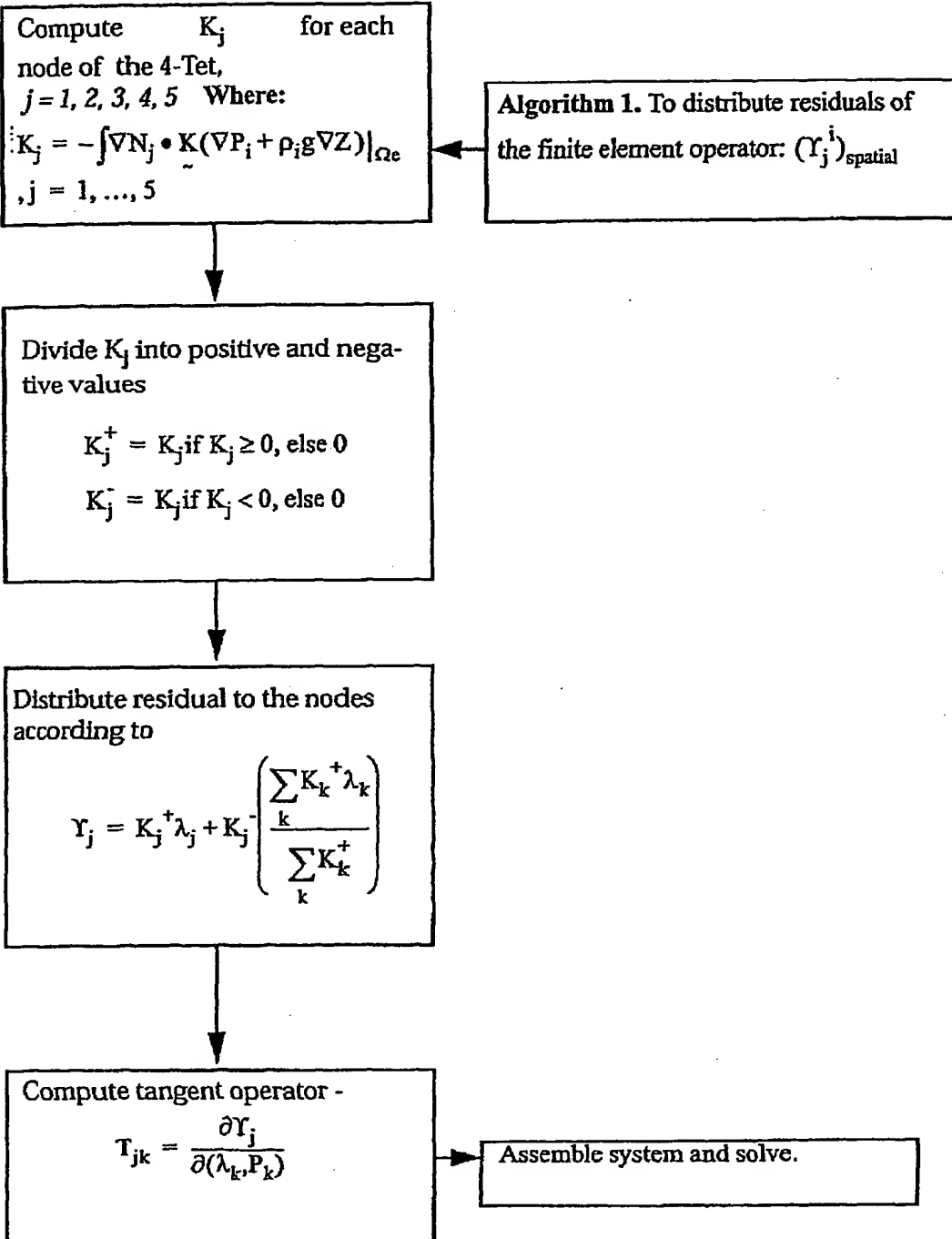
Compute K_j for each node of the 4-Tet,
$j = 1, 2, 3, 4, 5$ Where:
$$K_j = -\int \nabla N_j \bullet K(\nabla P_i + \rho_i g \nabla Z)\big|_{\Omega e}$$
$, j = 1, ..., 5$

Algorithm 1. To distribute residuals of the finite element operator: $(\Upsilon_j^i)_{spatial}$

Divide K_j into positive and negative values

$$K_j^+ = K_j \text{ if } K_j \geq 0, \text{ else } 0$$

$$K_j^- = K_j \text{ if } K_j < 0, \text{ else } 0$$

Distribute residual to the nodes according to

$$\Upsilon_j = K_j^+ \lambda_j + K_j^- \left(\frac{\sum_k K_k^+ \lambda_k}{\sum_k K_k^+} \right)$$

Compute tangent operator -
$$T_{jk} = \frac{\partial \Upsilon_j}{\partial (\lambda_k, P_k)}$$

Assemble system and solve.

Fig. 8

A. CLASSIFICATION OF SUBJECT MATTER

IPC 7   G01V11/00

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

IPC 7   G01V   G06F   E21B

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

EPO-Internal, COMPENDEX, WPI Data

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category°	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	WO 99 40532 A (EXXON PRODUCTION RESEARCH CO) 12 August 1999 (1999-08-12) page 11, line 6 -page 12, line 21	1,12,13
A	FARKAS EVA: "Linearization techniques of reservoir simulation equations: Fully implicit cases" PROCEEDINGS OF THE 1997 SPE RESERVOIR SIMULATION SYMPOSIUM;DALLAS, TX, USA JUN 8-11 1997, 1997, pages 87-95, XP000910556 Richardson, TX, USA the whole document	1,12,13

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[X] Further documents are listed in the continuation of box C.   [X] Patent family members are listed in annex.

° Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance

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"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
25 February 2002	11/03/2002

Name and mailing address of the ISA	Authorized officer
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Form PCT/ISA/210 (second sheet) (July 1992)

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication,where appropriate, of the relevant passages	Relevant to claim No.
A	FARKAS EVA: "Linearization techniques of reservoir-simulation equations: Fully implicit cases" RICHARDSON, TX, USA, vol. 3, no. 4, December 1998 (1998-12), pages 316-323, XP002189957 the whole document ---	1,12,13
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			NO	20003947 A	04-08-2000
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			EP	0995159 A1	26-04-2000
			FR	2775095 A1	20-08-1999
			WO	9942937 A1	26-08-1999
			NO	994843 A	09-11-1999
US 5710726	A	20-01-1998	NONE		